(a) (i) $AC = CD$ (given)

$\therefore \triangle ADC$ is isosceles

$\therefore \angle ADC = \angle DAC$ (base $\angle$'s of an isosceles triangle are equal)

$\therefore 2\angle ADC = \pi - \frac{\pi}{5}$ (angle sum of a $\triangle$)

$= \frac{4\pi}{5}$

$\therefore \angle ADC = \frac{\frac{4\pi}{5}}{2}$

$= \frac{2\pi}{5}$

$\therefore \angle ADC = 2\theta$ ($\frac{2\pi}{5}$ is $2 \times \frac{\pi}{5}$)

$\boxed{\begin{array}{l} \angle ADC = \angle ABD + \angle BAD \text{ (exterior angle = two opposite interior } \angle\text{'s)} \\ \therefore 2\theta = \theta + \angle BAD \\ \therefore \angle BAD = \theta \end{array}}$

In $\triangle$'s DBA and ABC

1. $\angle ABC$ is common

2. $\angle BAD = \angle ACB = \theta$ (proven $\nearrow$)

$\therefore \triangle DBA \,|||\, \triangle ABC$ (equiangular)

(ii) $\frac{BC}{BA} = \frac{AC}{BD}$ (corresponding sides in the same ratio, $\Delta DBA ||| \Delta ABC$)

$\therefore \frac{x}{1} = \frac{1}{x-1}$

$\therefore x(x-1) = 1$

$x^2 - x = 1$

$\therefore x^2 - x - 1 = 0$

(iii) $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Now, from (ii), $x^2 - x - 1 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

But $x > 0$, $\therefore x = \frac{1+\sqrt{5}}{2}$

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$\therefore \cos\frac{\pi}{5} = \frac{1^2 + x^2 - 1^2}{2 \times x \times 1}$

$= \frac{x}{2x}$

$\therefore \cos\frac{\pi}{6} = \frac{\frac{1+\sqrt{5}}{2}}{2 \times \frac{1+\sqrt{5}}{2}}$

$= \frac{1+\sqrt{5}}{4}$

(b) (i) when $t = 0$

$\frac{dV}{dt} = 2e^0 + 2e^{-0}$

$= 4$

$\therefore$ initial rate is $4L/s^{-1}$

(ii) $\int 2e^t + 2e^{-t}\, dt$

$\therefore V = 2e^t - 2e^{-t} + c$

(iii) when $V = 3$

$3 = 2e^t - 2e^{-t}$