7ai) The series has a limiting sum because each term becomes progressively smaller (≤ 1) but is still > 0.

ii) $S_\infty = \dfrac{1}{\sqrt{5}-3}$

$= \dfrac{1}{\sqrt{5}-3} \times \dfrac{\sqrt{5}+3}{\sqrt{5}+3}$

$= \dfrac{\sqrt{5}+3}{5-9}$ $= \dfrac{-\sqrt{5}-3}{4}$

bi) $t=0$, $V = 25(1)^2$

$V = 25$

ii) $\dfrac{25}{4} = 25\left(1-\dfrac{t}{60}\right)^2$

$1 - \dfrac{t}{30} + \dfrac{t^2}{3600} = \dfrac{1}{4}$

$t^2 - 120t + 2700 = 0$

$t^2 - 30t - 90t + 2700 = 0$

$t(t-30) - 90(t-30) = 0$

$(t-30)(t-90) = 0$

P 2700
S -120
F -30, -90

$\therefore t = 30, 90$

$\therefore$ cooler is $\frac{1}{4}$ full at 30 seconds

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iii) $\frac{dV}{dt} = -\frac{50}{60}\left(1-\frac{t}{60}\right)^2$

$= -\frac{5}{6}\left(1-\frac{t}{60}\right)$

at $t = 30$,

$\frac{dV}{dt} = -\frac{5}{6}\left(1-\frac{1}{2}\right)$

$= -\frac{5}{12}$

c i) The probability of getting matching socks is:

$P(RR) + P(WW) + P(YY) + P(BB)$

$= \left(\frac{1}{8}+\frac{1}{2}\right)^4 + \left(\frac{15}{56}\right)^4 + \left(\frac{50625}{9834496}\right)$