Question 7

a) i) The geometric series has a limiting sum as the common ratio is $(\sqrt{5}-2)$ and, thus, it is not possible for the series to continue infinitely

ii) limiting sum $= \dfrac{a}{1-r}$

$a = 1$

$r = (\sqrt{5}-2)$

limiting sum $= \dfrac{1}{1-(\sqrt{5}-2)}$

$= \dfrac{1}{1-\sqrt{5}+2}$

$= \dfrac{1}{(3-\sqrt{5})} \times \dfrac{(3+\sqrt{5})}{(3+\sqrt{5})}$

$= \dfrac{3+\sqrt{5}}{9-5}$

$= \dfrac{3+\sqrt{5}}{4}$

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b) i) $V = 25\left(1 - \frac{t}{60}\right)^2 \quad 0 \le t \le 60$

when $t=0$ $V = 25\left(1 - \frac{0}{60}\right)^2$

$= 25\ (1)^2$

$= 25$ litres

ii) $\frac{25}{4} = \left[25\left(1 - \frac{t}{60}\right)^2\right]$

$\frac{1}{4} = \left(1 - \frac{t}{60}\right)^2$

$\frac{1}{2} = 1 - \frac{t}{60}$

$-\frac{1}{2} = -\frac{t}{60}$

$-60 = -2t$

$\therefore t = 30$ seconds.

$\therefore$ The cooler was one quarter full after 30 seconds.

iii) $V = 25\left(1 - \frac{t}{60}\right)^2$

let $V = 25u^2$ where $u = 1 - \frac{t}{60}$

$\frac{dV}{du} = 50u \qquad \frac{du}{dt} = -\frac{1}{60}$

$\frac{dV}{dt} = 50\left(1 - \frac{t}{60}\right) \times -\frac{1}{60}$

$$= -\frac{50}{60}\left(1 - \frac{t}{60}\right)$$

when $t = 30$ $\quad \frac{dV}{dt} = -\frac{50}{60}\left(1 - \frac{30}{60}\right)$

$$= -\frac{50}{60}\left(1 - \frac{1}{2}\right)$$

$$= -\frac{50}{60} \times \frac{1}{2}$$

$$= \frac{-50}{120}$$

$$= \frac{-5}{12}$$

$$= \frac{50t}{3600} - \frac{50}{60}$$

when $t = 30$ $\quad \frac{dV}{dt} = \frac{1500}{3600} - \frac{50}{60}$

$$= -\frac{5}{12}$$

$\therefore$ The water was draining at $\frac{5}{12}$ litres/second

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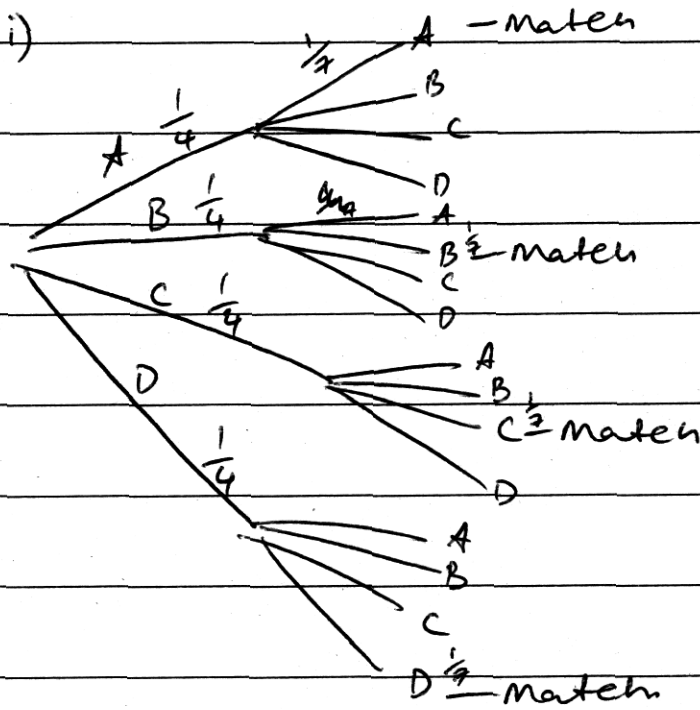
c) A B C D

i)



P(not having a matching pair) = 1 − P(having a matching pair)

$= 1 - 4\left(\frac{1}{4} \times \frac{1}{7}\right)$

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$ as required.

ii) P(not having a matching pair after third sock)

~~= P(not having a matching pair after second sock)~~

$= \frac{1}{4} \times \frac{6}{7} \times \frac{2}{3}$

$= \frac{1}{7}$

iii) P(First three socks include matching pair)

$= 1 - P$(not having a matching pair after third sock)

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$