a) $r = \sqrt{5} - 2$

$\therefore$

$0 < r < 1$

$\therefore$ the series has a limiting sum

ii) $S = \dfrac{1}{1-(\sqrt{5}-2)}$

$= \dfrac{1}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \dfrac{3+\sqrt{5}}{4}$

$= \dfrac{1}{\sqrt{5}+3}$

$= \dfrac{1}{\sqrt{5}+3} \times \dfrac{-\sqrt{5}-3}{-\sqrt{5}-3}$

$= \dfrac{-\sqrt{5}-3}{5-9}$

$= \dfrac{-\sqrt{5}-3}{-4}$

$= \dfrac{\sqrt{5}+3}{4}$

$S_{100} = \dfrac{1((\sqrt{5}-2)^{100}-1)}{(\sqrt{5}-2)-1}$

b) i) let $t = 0$

$V = 25(1)^2$

$= 25$ litres

ii) $\dfrac{25}{100} = 25\left(1-\dfrac{t}{60}\right)^2$

$\dfrac{25}{100} = 1 - \dfrac{t}{30} + \dfrac{t^2}{3600}$

$900 = 3600 - 120t + t^2$

$t^2 - 120t + 2700 = 0$

$t = \frac{120 \pm \sqrt{120^2 - 4.2700}}{2}$

$= \frac{120 \pm 60}{2}$

$= 90$ and 30 seconds

$0 \leq t < 80$

$\therefore$ when $t = 30$ seconds

iii) $V' = 25.2\left(1 - \frac{t}{60}\right).\frac{-1}{60}$ $\quad V' = 2t - 120$

$= -\frac{5}{6}\left(-\frac{t}{60}\right)$

$= \frac{t}{72} - \frac{5}{6}$

$V = 25\left(1 - \frac{t}{30} + \frac{t^2}{3600}\right)$

$= 25 - \frac{5t}{6} + \frac{5t^2}{72}$

$V' = -\frac{5}{6} + \frac{10t}{72}$

$= \frac{5t}{30} - \frac{5}{6}$

$\frac{5t}{30} = \frac{5}{6}$

$30t = 150$

$t = 5$

$V'' = 25\left(1 - \frac{2t}{60} + \frac{t^2}{3600}\right)$

$= 25 - \frac{5t}{6} + \frac{t^2}{144}$

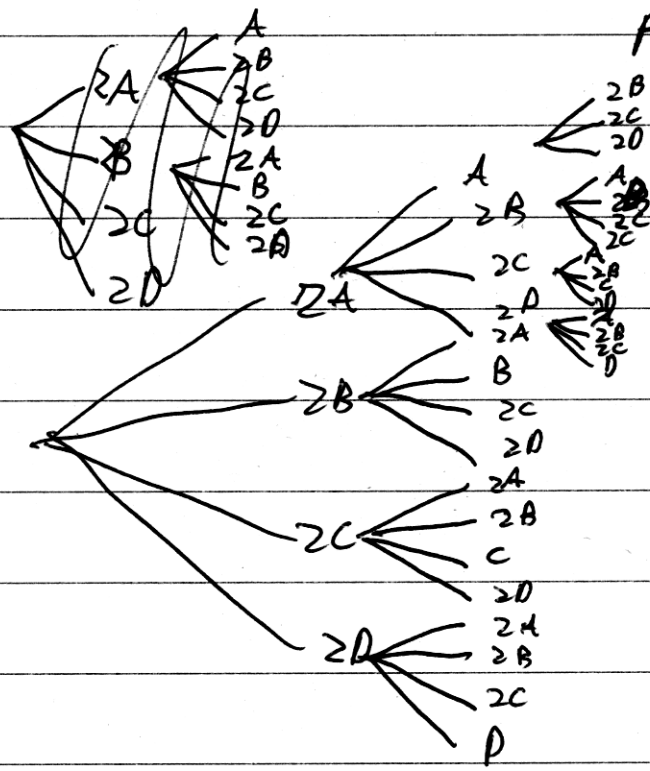
$V' = \frac{t}{72} - \frac{5}{6}$

sub in $t = 30$

$V' = \frac{30}{72} - \frac{5}{6}$

$= -\frac{5}{12}$ L/second

c) let the doors be A, B, C & D



probability of selecting 2 × A
$= 1/28$

$2 \times B = 1/28$

$2 \times C = 1/28$

$2 \times D = 1/28$

∴ probability of any match
$= \frac{4}{7}$

∴ probability of no match
$= \frac{3}{7}$

ii) probability of selecting 2×A

$= \frac{9}{96}$

$\therefore$ $2 \times B = \frac{9}{96}$

$2 \times C = \frac{9}{96}$

$2 \times D = \frac{9}{96}$

$\therefore$ selecting a pair is $= \frac{3}{8}$

$\therefore$ probability of no pair is $= \frac{5}{8}$

iii) from above $= \frac{3}{8}$