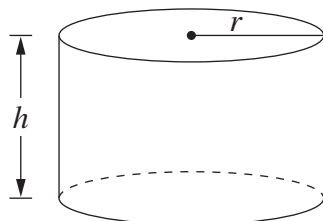


**Question 5** (12 marks) Use the Question 5 Writing Booklet.

- (a) A rainwater tank is to be designed in the shape of a cylinder with radius  $r$  metres and height  $h$  metres.



The volume of the tank is to be 10 cubic metres. Let  $A$  be the surface area of the tank, including its top and base, in square metres.

- (i) Given that  $A = 2\pi r^2 + 2\pi rh$ , show that  $A = 2\pi r^2 + \frac{20}{r}$ . 2
- (ii) Show that  $A$  has a minimum value and find the value of  $r$  for which the minimum occurs. 3
- (b) (i) Prove that 1
- $$\sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}.$$
- (ii) Hence prove that 1
- $$\sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}.$$
- (iii) Hence use the table of standard integrals to find the exact value of 2

$$\int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x} dx.$$

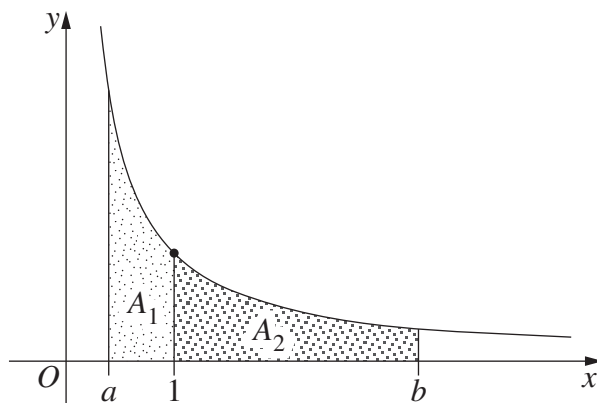
**Question 5 continues on page 9**

## Question 5 (continued)

- (c) The diagram shows the curve  $y = \frac{1}{x}$ , for  $x > 0$ .

3

The area under the curve between  $x = a$  and  $x = 1$  is  $A_1$ . The area under the curve between  $x = 1$  and  $x = b$  is  $A_2$ .



The areas  $A_1$  and  $A_2$  are each equal to 1 square unit.

Find the values of  $a$  and  $b$ .

**End of Question 5**