

Start here for

Question Number:

10

(a) In $\triangle ABC$ and $\triangle ACD$.

$$\angle CAD = \angle BAC \quad (\text{common})$$

$(a+y) = AB$, $\therefore AD = a$ (same ratio, as a is drawn from AB).

AC is (common hence in the same ratio.)

$\therefore \triangle ABC$ is similar to $\triangle ACD$ (equal included angle and 2 sides in same ratio)

(i) $a^2 = b^2 + c^2 - 2bc \cos A$

let $a = x$, $b = a$, and $c = ay$.

~~(ii)~~ $x^2 = a^2 + y^2 - 2ay \cos x$

(iii) $y = a(1 - \cos \theta)$

Subst

(iv) $y = ay - a$

$$ay = x^2 - a^2$$

$$y = \frac{x^2 - a^2}{a} - a$$

$$y = a(x^2 - a - 1)$$

Now $y \leq$

$$(b) \quad v = \int_a^b y^2 dx.$$

$$V \text{ for a cone sphere is } \frac{4}{3} \pi r^3 \div 4$$

$$\int_0^r (r^2 - x^2) dx = \frac{\pi r^3}{3}$$

A of total sphere. $\frac{r^2 x - \frac{x^3}{3}}{3} \div 4$

~~$$\frac{r^2 x - \frac{x^3}{3}}{3}$$~~

~~$$\left(\frac{r^2 x - \frac{x^3}{3}}{3} \right) \div 4$$~~

~~$$\frac{3r^2 x - x^3}{3} \times \frac{1}{4}$$~~

Total area for quarter. $\left[\frac{3r^2 x - x^3}{12} \right]_0^r$

Now for

$$= \frac{-3r^3 r - r^3}{12}$$

$$\frac{-3r^4 - r^3}{12}$$

$$\frac{\pi r^3}{3} \left(2 - \frac{1}{2} r^2 \sin^2 \theta \right)$$

$$\text{Area of quarter} = \frac{4}{3} \pi r^3 \div 4$$

$$= \frac{\pi r^3}{3} - \text{area of } \triangle POA - \text{area of segment.}$$

$$\frac{\pi r^3}{3} - \left(\frac{1}{2} r^2 \sin \theta + \frac{r^2 \sin^3 \theta}{3} \right)$$
~~$$\frac{r^2 \sin^3 \theta}{3}$$~~

Additional writing space on back page.

$$(ii) \textcircled{1} \quad L = r\theta$$

$$V = \frac{4}{3} \pi r^3$$

$$r = \theta/L$$

$$\frac{1}{2} = \frac{4}{3} \pi r^3$$

$$L = 1/2$$

$$\frac{1}{2} = \frac{4}{3} \pi \frac{\theta}{L}$$

$$\frac{1}{2} = \frac{4}{3} \pi \frac{\theta}{2}$$

$$\frac{8}{3} \pi \theta = 1/2$$

$$\theta = 1/2$$

$$\frac{3}{16} \pi$$

$$\textcircled{2} \quad \underline{V_3}$$

$$\underline{\theta = \frac{3}{16} \pi}$$

