Start here for Question Number: **10**

a) i) $\angle ACD = \angle CAD$ (base $\angle$'s of isos. Δ)

$\therefore$ $180 - \cancel{2}\theta = 2\angle CAD$ ($\angle$ sum of Δ)

$\angle \cancel{B}AD = 90 - \frac{\theta}{2}$

$\angle CAD = \angle CBA$ (Base $\angle$'s of isos. ΔABC)

$\angle ACD = 180 - 2(90 - \frac{\theta}{2})$ $\angle$ sum of Δ)

$\therefore \angle ACD = \theta$

$\therefore \Delta ABC \,|||\, \Delta ACD$ (equiangular)

ii) $\frac{x}{a}$ $\cancel{(a+y)}$ $\frac{a+y}{x} = \frac{x}{a}$ (corresponding sides of ~~equal~~ similar Δ's are in equal ratio)

$a(a+y) = x^2$

$\therefore x^2 = a^2 + ay$

iii) a^2 $\frac{x^2 - a^2}{a} = y$

$\frac{(x+a)(x-a)}{a} = y$

~~Dsq~~ $(a+y)^2 = x^2 + x^2 - 2x^2 \cos\theta$

$(a+y)^2 = 2x^2 - 2x^2 \cos\theta$

$= 2x^2(1 - \cos\theta)$

$(a+y^2)$

$(a+y^2) = 2(a^2 + ay)(1 - \cos\theta)$

$a^2 + 2ay + y^2 = 2a^2 + 2ay(1 - \cos\theta)$

$\cancel{a^2} + \cancel{a^2} + \cancel{2ay} + y^2 = \cancel{2}a^2 + \cancel{2y}$ ~~...~~ $\cos\theta - 2y\cos\theta$

$y^2 = a^2 - 2a^2\cos\theta - 2ay\cos\theta$

$\cancel{\text{II}}$

(ii) $\frac{a+y}{\sin\theta} = \frac{x}{\sin(60-\theta)}$

$\cancel{a+y = x}$ $\cancel{x}$

$(a+y)^2 = x^2 + x^2 - 2x^2 \cos\theta$

$= x^2(2 - 2\cos\theta)$

$a^2 + 2ay + y^2 =$

(iv) $\cancel{y \geq}$ $y = a(1 - 2\cos\theta)$

max value of $1 - 2\cos\theta = 3$

$\therefore y \leq 3a$

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b) $x^2 + y^2 = r^2$

i) $\cancel{x^2 - r^2}$ $r^2 - x^2 = y^2$

$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

$\sin\theta = \frac{OA}{r}$

$r\sin\theta = OA$

$V = \frac{2}{3}\pi r^2 - \pi\int_0^{r\sin\theta} x^2 - r^2 \, dx$

$= \frac{2}{3}\pi r^3 - \pi\left[-\frac{x^3}{3} + r^2 x\right]_0^{r\sin\theta}$

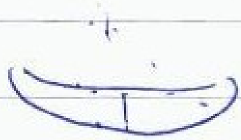
$= -\pi\left[\left(-\frac{r^3\sin^3\theta}{3} + r^3\sin\theta\right) - (0 - 0)\right]$

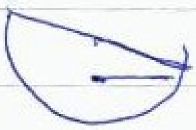
$= \frac{2\pi r^3}{3} + \frac{\pi r^3\sin^3\theta}{3} - \frac{3r^3\sin\theta}{3}$

$= \frac{\pi r^3}{3}\left[2 + \sin^3\theta - 3\sin\theta\right]$

$= \frac{\pi r^3}{3}\left[2 - 3\sin\theta + \sin^3\theta\right]$

ii) (1)





$OB = r$

$OA = r\sin\theta$

$h = r - r\sin\theta$

$h = r(1 - \sin\theta)$

Start here.

ii)

(2)

~~is~~ & if $h = r(1-\sin\theta)$

new r is $\cancel{\sqrt{}}$

$\cancel{\sin\theta = \frac{r\sin\theta}{new}}$

$r^2 - (r\sin\theta)^2$

$= r^2 - r^2\sin^2\theta$

$rN = r^2(1-\sin^2\theta)$

$V = \cancel{\tfrac{1}{2}}\ \frac{4}{3}\pi r^2$

$\frac{2}{3}\pi r^3$

-1

fraction left $= \left(\dfrac{\frac{2}{3}\pi r^3}{\frac{\pi r^3}{3}(2-3\sin\theta+\sin^3\theta)}\right)^{-1}$

$\dfrac{\text{new vol}}{\text{original vol}} = \dfrac{(i)}{\frac{1}{2}\frac{4}{3}\pi r^3} = \left(\dfrac{2}{2-3\sin\theta+\sin^3\theta}\right)^{-1}$ left.

$= \dfrac{(i)}{\frac{2\pi r^3}{3}} = \dfrac{2-3\sin\theta+\sin^3\theta}{2}$