

Start here for  
Question Number: **5**

Q5.a)

i).  $A = 2\pi r^2 + 2\pi r h$

~~$A = 2\pi r^2$~~

~~$2\pi r h = \text{surface area.}$~~

~~$r = \frac{2\pi}{h}$~~

~~$h = 2\pi$~~

$V = \pi r^2 h$

~~$20$~~   $= \pi r^2 h$

$h = \frac{20/\pi}{r^2}$

$A = 2\pi r^2 + 2\pi r \left(\frac{20}{\pi r^2}\right)$

$= 2\pi r^2 + 2 \frac{20}{r}$

$= 2\pi r^2 + \frac{20}{r}$

ii).  $A = 2\pi r^2 + 2\pi r h$

$\frac{dA}{dr} = 4\pi r + 2\pi h$

$A = 2\pi r^2 + \frac{20}{r}$

$\frac{dA}{dr} = 4\pi r - 20r^{-2}$

$= 4\pi r - 20r^{-2}$

$\frac{dA}{dr} = 0$

$4\pi r - 20r^{-2} = 0$

let  $r = \sqrt[2]{\frac{20}{4\pi}}$

$4\pi r = 20r^{-2}$

$\frac{4\pi}{20} = \frac{r^{-2}}{r}$

$\frac{\pi}{5} = r$

$\frac{\pi}{20} > 0$   $\therefore$  minimum value

$\therefore r = \frac{\pi}{20}$

$$b) i) \quad \sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$$

$$\text{LHS} = \sec^2 x + \sec x \tan x =$$

$$1 + \cos^2 x + \frac{1}{\cos x} \times \frac{\sin x}{\cos x} =$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$ii) \quad \sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$$

$$\frac{1 + \sin x}{\cos^2 x} =$$

$$iii) \quad \int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x} dx$$

$$= \left[ \frac{1}{1 - \sin x} \right]_0^{\frac{\pi}{4}}$$

$$= \left( \frac{1}{1 - \sin \frac{\pi}{4}} - \frac{1}{1 - \sin 0} \right)$$

$$= \left( \frac{1}{1 - \frac{1}{\sqrt{2}}} - 1 \right)$$

$$= \frac{1}{1 - \frac{1}{\sqrt{2}}} - 1$$

$$\left. \begin{array}{l} \sqrt{2} \\ 45 \\ 1 \end{array} \right\}$$

Additional writing space on back page.

$$c) \quad y = \frac{1}{x} \quad \text{for } x > 0$$

$$\int_1^a \frac{1}{x} dx$$

$$1 = [\log_e x]_1^a$$

$$1 = \log_e a + \log_e 1$$

$$-\log_e a = -1 + \log_e 1$$

$$-\log_e a = -1$$

$$\log_e a = 1 \quad a = e^1$$

$$\int_b^1 \frac{1}{x} dx$$

$$1 = [\log_e x]_b^1$$

$$1 = \log_e 1 - \log_e b$$

$$\log_e b = \log_e 1 - 1$$

$$\log_e b = -1$$

$$b = e^{-1}$$

