Start here for Question Number: 5

a) Volume = $10\ m^3$
Surface Area = A. incl. Top & Base

i) $A = 2\pi r^2 + 2\pi rh$.
~~$V = \pi r^2 \times h$.~~
~~$\pi r^2 h = 10$~~
~~$h = \frac{10}{\pi r^2}$~~
~~$= \frac{5}{\pi r^2}$~~

$V = \pi r^2 \times h$
~~There~~. $\therefore \pi r^2 h = 10$
$h = \frac{10}{\pi r^2}$

Sub h into A $\to$ $2\pi r^2 + 2\pi r \times \frac{10}{\pi r^2}$
$\therefore A = 2\pi r^2 + \frac{20\cancel{\pi}\cancel{r}}{\cancel{\pi} r^{\cancel{2}}}$
$= 2\pi r^2 + \frac{20}{r}$

ii) for min or max $\frac{dA}{dr} = 0$.
$\frac{dA}{dr} = 4\pi r + (-20)r^{-2}$
$= 4\pi r - \frac{20}{r^2}$
$4\pi r - \frac{20}{r^2} = 0$
$4\pi r^3 - 20 = 0$
$\pi r^3 - 5 = 0$
$\pi r^3 = 5$
$r^3 = \frac{5}{\pi}$
$\therefore r = \sqrt[3]{\frac{5}{\pi}}$

r	$\sqrt[3]{\frac{5}{\pi}} - 0.1$	$\sqrt[3]{\frac{5}{\pi}}$	$\sqrt[3]{\frac{5}{\pi}} + 0.1$
A	$(-)$	0	$(+)$

$\therefore$ There is a min value

$\therefore$ Min value is 1.16

b) i) Pf: $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$.

$LHS = \sec^2 x + \sec x \tan x$.

$= \dfrac{1}{\cos^2 x} + \dfrac{1}{\cos} \times \dfrac{\sin x}{\cos x}$.

$= \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos^2 x}$

$= RHS$

Pf $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$

ii) $LHS = \sec^2 x + \sec x \tan x$

$= \dfrac{1+\sin x}{\cos^2 x}$ → fm i)

$= \dfrac{1+\sin x}{1-\sin^2 x}$ → fm $\sin^2 x + \cos^2 x = 1$.

$= \dfrac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}}$

$= \dfrac{1}{1-\sin x}$

$= RHS$

iii) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx$.

$= \displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x + \sec x \tan x \,dx$.

$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{4}}$

$= \left(\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right) - \left(\tan 0 + \sec 0\right)$

$= \left(1 + \frac{\sqrt{2}}{2}\right) - (0 + 1)$

$= \frac{\sqrt{2}}{2}$ //.

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c) $y = \frac{1}{x} \qquad x > 0$

$~~\int_a^b \frac{1}{x}\,dx~~$

$~~= [\ln x]_a^b~~$

$~~= \ln b -~~ \ln a$

$\int_a^1 \frac{1}{x}\,dx = A_1$

$= [\ln x]_a^1$

$= \ln 1 \cancel{+} \ln a$

$= \ln a$

then $\ln a = 1$ (given)

$e^{-\ln a} = e^1$

$\frac{1}{a} = e$

$\therefore a = \frac{1}{e}$ or 0.3679 (4dp)

$\int_1^b \frac{1}{x}\,dx = A_2$

$= [\ln x]_1^b$

$= \ln b - \ln 1$

$= \ln b$

then $\ln b = 1$ (given)

$e^{\ln b} = e^1$

$\therefore b = e$ or 2.7183 (4dp)